Math 157 – Calculus II Final Exam – Spring 2024

April 30, 2024

SHOW ALL WORK. Justify your answers! Simplify your answers. Give step-by-step explanations to get credit for answers. Give EXACT answers whenever possible. Solve all parts of any 10 out of the 15 problems below. Each of the 15 problems = 20 points. Exam total = 200 points.

1. (a) Find the area bounded by the curves $y = x^2 - 1$ and $y = 2x + 7$.

   (b) Find the average value f_{avg} of $f(x) = \dfrac{1}{x^2}$ on the interval $[1, 3]$ and c in the given interval such that $f_{\text{avg}} = f(c)$.

2. Let R be the region in the first quadrant below the curve $y = \sqrt{x}$ from $x = 1$ to $x = 2$. Compute the volume of the solid obtained by rotating R:

   (a) about the x-axis; (b) about the y-axis.

3. Evaluate the integrals:

   (a) $\displaystyle\int_0^{2\pi} x^2 \sin(2x)\, dx\,;$ (b) $\displaystyle\int e^{3x} \cos x\, dx\,.$

4. Determine whether the integral is convergent or divergent. Evaluate the integrals that are convergent.

   (a) $\displaystyle\int_1^{\infty} \frac{x}{e^{2x}}\, dx\,;$ (b) $\displaystyle\int_3^{\infty} \frac{1}{(x-2)^{3/2}}\, dx\,.$

5. Evaluate the integrals:

   (a) $\displaystyle\int \sec x \tan^3 x\, dx\,;$ (b) $\displaystyle\int \sin^2 x \cos^3 x\, dx\,.$

6. Evaluate the integrals:

   (a) $\displaystyle\int \frac{1}{(x^2+9)^{3/2}}\, dx\,;$ (b) $\displaystyle\int \frac{2x+3}{x^2-4}\, dx\,.$

7. (a) Write the partial fractions decomposition of $\dfrac{10}{(x-1)(x^2+9)}\,.$

   (b) Evaluate the integral $\displaystyle\int \frac{10}{(x-1)(x^2+9)}\, dx\,.$

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8. (a) Find an equation of the line tangent to the curve given by $x = 2 + \ln t$, $y = t^2 - 3$ at the point $(2, -2)$.

   (b) Find the length of the curve defined by $x = -\sin^3 t, y = -\cos^3 t$ over the interval $0 \le t \le \frac{\pi}{2}$.

9. Compute the surface area of the surface obtained by rotating the curve given by $y = x^3$ from $x = 0$ to $x = 1$ about the x-axis.

10. Determine whether each of the following series converges conditionally, converges absolutely, or diverges. Remember to justify your answers.

    (a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n^3}{n!}$;

    (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2 + 3n - 2}{3n^2 + 5n + 1}$;

    (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1}$;

    (d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \arctan n}{n^2}$.

11. (a) Graph the curve $r = 2(1 + \cos\theta)$.

    (b) Find the area of the region in the plane enclosed by the curve $r = 2(1 + \cos\theta)$.

12. Find the interval and radius of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(5x-4)^n}{n^3}$.

13. A spring has a natural length of 40 cm. If a 60 N force is needed to keep the spring compressed 10 cm,

    (a) how much work is done during this compression?

    (b) how much work is required to compress the spring to a length of 25 cm?

    Hint: Recall that Hooke's Law says that the force needed to keep a spring compressed a distance x beyond its natural length is kx, where k is the spring constant of the spring.

14. Consider the function $f(x) = \sin(x)$.

    (a) Write the degree three Taylor polynomial $T_3(x)$, centered at $x = 0$, for this $f(x)$.

    (b) Use your answer in part (a) to give an estimate for the value of $f(-1)$.

    (c) Give an upper bound on the error for your estimate from part (b). *Hint:* Recall that the Taylor series for $\sin x$ at $x = 0$ is alternating.

15. (a) Approximate $\displaystyle\int_{-2}^{4} (x+1)^2\, dx$ by using the midpoint rule with $n = 3$ subintervals.

    (b) What is the error of your approximation compared to the true value of this definite integral?